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Numerical impact study of bonus distribution on default

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Introduction

- With-Profit Products
- Policyholder sharing the profits
- Two types of Bonuses:
  - Reversionary Bonus
  - Terminal Bonus

Introduction

- Traditional Bonus Distribution Strategy
  - Asset Share Study
  - Comparison of Policy Value against Life Par Fund
  - Asset of Life Par Fund is sufficient
  - Following the company's specific bonus principles

Introduction

- Pros and Cons
  - Common method based on past actual experience
  - Future projection based on deterministic assumptions, no stochastic simulation
  - Insufficient information to management
  - No management decision

Objectives

- Modeling bonus declaration
- Consideration of Singapore Regulation
- Quantifying impact of management decision on the default probability
- Incorporation of future long term investment rate and bonus rate

Scope

- Single Premium with profit Endowment for 20 years policy term with guaranteed yield
- No expense charges, lapses and mortality
- Bonus comes from the surplus

Model

- Steffensen **(2004)**
  - Participating life insurance
  - Positive dividends
  - Linear dividends are optimal adopting a power utility optimization criterion

Model

- Balance Sheet at time T

ASSET	Liability
Asset(T)	Policy Value (T)
	Allocated Bonus (T)
	Free Capital (T)

Model

- Asset
  - Market value of insurer’s asset
  - Ignorance of asset mix
  - Geometric Brownian motion

Model

- Liability
  - Policy Value
  - Allocated bonus
  - Free capital

Base Model

- Bonus Rate and Policy Value – Model 1

$$r_b = Max(r_g, \alpha(\frac{B(T-1)}{P(T-1)} - \gamma)$$

$$P(T) = P_0 \prod_{i=1}^{i=T}(1 + Max(r_g, \alpha(\frac{B(i-1)}{P(i-1)} - \gamma)))$$

- α is distribution factor reflecting management decision
- γ is a buffer which is linked to solvency requirement
- r_g is guaranteed return

Model 2

- Minimum Condition Liability (MCL)

$$MCL(T) = [(1 + \mathrm{r_g} + \mathrm{r_1}) \cdots (1 + \mathrm{r_g} + \mathrm{r_{T-1}})] \times E_T^r (1 + \mathrm{r_g})^{n-T}$$

  - Past declared guaranteed bonus only
  - Discounted using risk free rate

Model 2

- Policy Liability (PL)

$$PL(T) = [(1 + \mathrm{r_g} + \mathrm{r_1}) \cdots (1 + \mathrm{r_g} + \mathrm{r_{T-1}})] \times E_T^i (1 + \mathrm{r_g} + \mathrm{r_{T-1}})^{n-T}$$

  - Past declared guaranteed and future anticipated bonus
  - Discounted using best estimated long term investment returns

Model 2

- Bonus Rate and Policy Value – Model 2

$$r_b = Max(r_g, \alpha\left(\frac{(A(T-1)-\max(MCL(T-1),PL(T-1)))}{\max(MCL(T-1),PL(T-1))} - \gamma\right))$$

$$P(T) = P_0 \prod_{i=1}^{i=T}(1 + Max(r_g, \alpha(\frac{(A(i-1)-\max(MCL(i-1),PL(i-1))}{\max(MCL(i-1),PL(i-1))} - \gamma)))$$

Model 3

- PL calculation
  - Latest declared bonus rate
  - No future investment
  - Disregard future surplus
- "Future Anticipated Bonuses" in policy liability?

Model 3

- Future long term interest model

$$dr(t) = k(l - r(t))dt + \sigma\sqrt{r(t)}dW(t)$$

$$r_b^f = \max\left(a(r - r_{lv}), 0\right)$$

Model 3

- Include future anticipated bonuses r_b^f in policy liability
- Only credited when

  Simulated long term investment return >
  best estimated long term investment return

Traditional Bonus Strategy

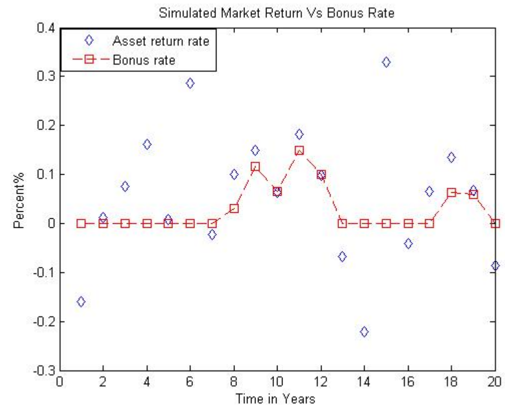
- With lapse and mortality
- 20,000 simulated asset paths (σ=0.15, long term investment=5%)
- Default probability = 57%
- Although long term investment return remain at the best estimated level, the fluctuation will lead company more probability to have insufficient asset to pay out the benefit

Numerical Results

- Assumptions
  - initial asset value =1
  - long term investment = 0.05
  - initial liability = 1, initial surplus = 0
  - guaranteed return rate for policy holder 0.01
  - No friction cost

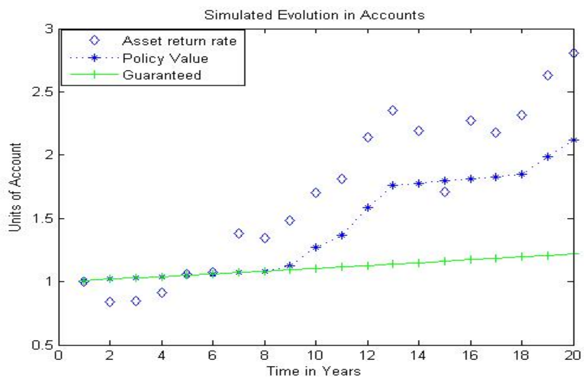
Numerical Results(base model)

Simulated Investment Return vs. Bonus Rate



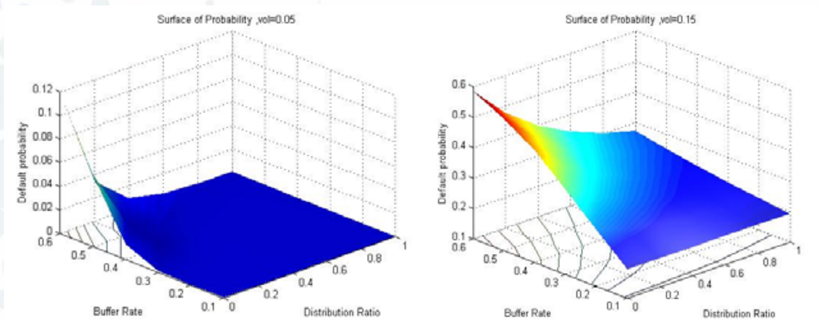
Numerical Results (base model)

Simulated evolution of the accounts



Numerical Results (base model)

Default Probability under different volatility



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Numerical Results (base model)

- Default probability
  - Asset volatility
  - Distribution ratio
  - Buffer ratio
- Asset volatility is critical
- Iso-default probability contour and flexibility of bonus crediting

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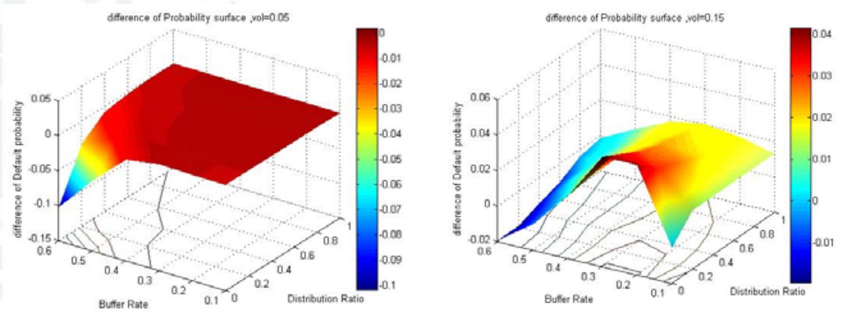
Redefining Risk • Creating Value

Numerical Results (Model 2)

- Difference of Default Probability between base model and model2



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Numerical Results (Model 2)

- Discounting rate in model 2 is risk free rate set by regulator
- Same result pattern as base model given volatility, distribution ratio and buffer ratio
- Significant relative difference of default probability between two models at low asset volatility
- The relative difference is decreasing when volatility rising

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Numerical Results (Model 2)

- Asset Volatility σ = 0.15
- Discounted by risk free rate for MCL

Distribution Ratio	Default Probability					
	Buffer Ratio					
	0.1	0.2	0.3	0.4	0.5	0.6
0.0	0.19080	0.19080	0.19080	0.19080	0.19080	0.19080
0.2	0.28895	0.25495	0.23425	0.21920	0.20865	0.20315
0.4	0.37925	0.31540	0.27260	0.24605	0.22885	0.21590
0.6	0.46410	0.38030	0.31745	0.27715	0.24925	0.23125
0.8	0.53195	0.43270	0.36005	0.30770	0.27395	0.25035
1.0	0.58070	0.47650	0.39745	0.33880	0.29680	0.26605

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Numerical Results (Model 2)

- Asset Volatility σ = 0.15
- Discounted by constant 5% for MCL

Distribution Ratio	Default Probability					
	Buffer Ratio					
	0.1	0.2	0.3	0.4	0.5	0.6
0.0	0.19175	0.19175	0.19175	0.19175	0.19175	0.19175
0.2	0.51660	0.46475	0.41965	0.38090	0.34920	0.32325
0.4	0.66975	0.60050	0.53935	0.48740	0.44150	0.40155
0.6	0.74975	0.67350	0.60535	0.54890	0.49980	0.45305
0.8	0.79740	0.72495	0.65690	0.59715	0.54225	0.49275
1.0	0.83010	0.75865	0.69760	0.63745	0.58365	0.53135

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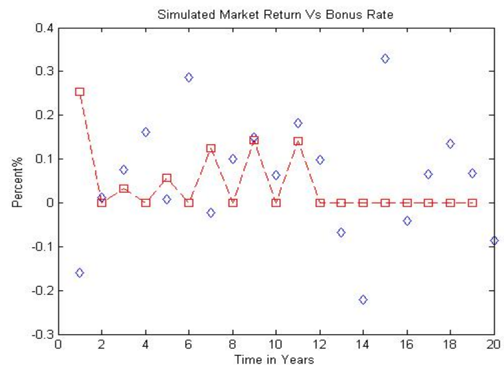
Numerical Results(model 2)

Simulated Investment Return vs. Bonus Rate



Numerical Results (Model 2)

- The default probability spikes using constant 5% as a discounting rate for MCL
  - More bonus rates are credited due to the discounting rate
- Review asset liability matching when risk free rate changes

Numerical Results(model 3)

- Short Rate Volatility σ = 0.085

Distribution Ratio	Default Probability: starting long term investment return rate 0.025	0.03	0.035	0.04	0.045	0.05
0.1	0.6550	0.6585	0.6645	0.6625	0.6640	0.6585
0.2	0.6545	0.6565	0.6640	0.6615	0.6630	0.6570
0.3	0.6520	0.6560	0.6630	0.6595	0.6610	0.6560
0.4	0.6525	0.6550	0.6620	0.6575	0.6605	0.6555
0.5	0.6525	0.6545	0.6610	0.6575	0.6585	0.6545
0.6	0.6520	0.6535	0.6605	0.6570	0.6580	0.6540

Numerical Results(model 3)

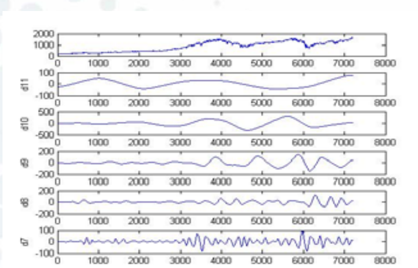
- Default probability insensitive to parameters
  - α
  - Initial long term investment return rate
- Further investigation is needed
  - Discounting factor
  - CIR~ $r_{T+i} = r_T + cP$ $c = \frac{(1-e^{-\kappa i})\sigma^2}{2k}$ $P \sim X^2(\frac{4kl}{\sigma^2}, 2kr_T e^{-ki})$

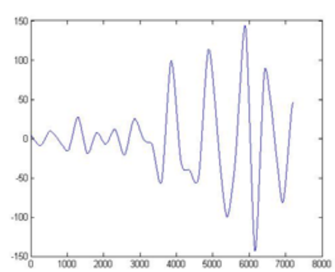
Numerical Results(model 3)

- more complex long term investment return can be tested through scenario setting e.g.
  - Financial cyclic
  - Taylor rule
- Financial cyclic detection
  - Wavelets or HP filter

Numerical Results(model 3)

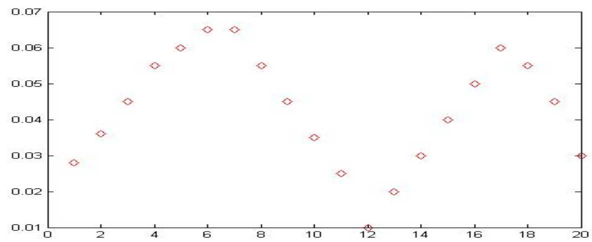
- Financial cyclic detection using wavelets





Numerical Results(model 3)

- Taylor rule
- Settled scenario



Conclusion

- Long term low level of interest, future macroeconomic and financial market volatility threatened the solvency of company
- Proposed framework is helpful for quantifying risk analysis and management decision
- Reduce of asset volatility significantly lower the default risk by proper portfolio allocation

Conclusion

- Partial flexible bonus rate decision by selecting proper control parameters
- Change of risk free rate may substantially increase default probability
- Integration of anticipation of future best estimated investment return

Future Task

- Further adaption of the models with variables of mortality and surrender
- Multiple policies
- Stochastic control to keep solvency target while maximize company or shareholders' profit

Q & A

- Thanks!